

# THE MOUNT STROMLO SATELLITE LASER RANGING (SLR) SYSTEM LOCAL TIE CONNECTIONS BEFORE AND AFTER THE 2003 DESTRUCTIVE CANBERRA FIRES

J. Dawson (1), G. Johnston (1), S. Naebkhil (1), R. Govind (1) and J. Luck (2)

(1) Geoscience Earth Monitoring, Geoscience Australia, Canberra, Australia.

(2) Electro Optic Systems Pty Limited, Queanbeyan, Australia.

John.Dawson@gc.gov.au Tel: +61 2 6249 9028

## Abstract

*The integrity and strengths of multi-technique terrestrial reference frames such as ITRF2000 depend on the precisely measured and expressed local tie connection between space geodetic observing systems at co-located observatories. The destructive Canberra fires of January 2003 completely destroyed the Mount Stromlo Satellite Laser Ranging observatory including the SLR, DORIS, GLONASS and GPS instruments located at the site. Fortunately, Geoscience Australia has routinely performed classical terrestrial surveys at Mount Stromlo, including surveys in 1999, 2002 and 2003 (post-fire). These surveys have included the determination of the SLR invariant point or IVP. Using existing undamaged survey pillars a consistent stable terrestrial network has been used to compute the relationship between the pre and post fire local tie connections. This relationship includes the millimetre level accurate connections and their associated variance covariance matrix and provides an unbroken contribution of the Mount Stromlo observatory to future terrestrial reference frames and other scientific outputs. Observational and analysis techniques are reviewed and results are given.*

## Introduction

The Mount Stromlo Satellite Laser Ranging (SLR) observatory is located in the Australian Capital Territory (ACT), see Figure 1. In this analysis the ground survey observations from the 1999, 2002 and 2003 surveys are subject to a combined analysis.

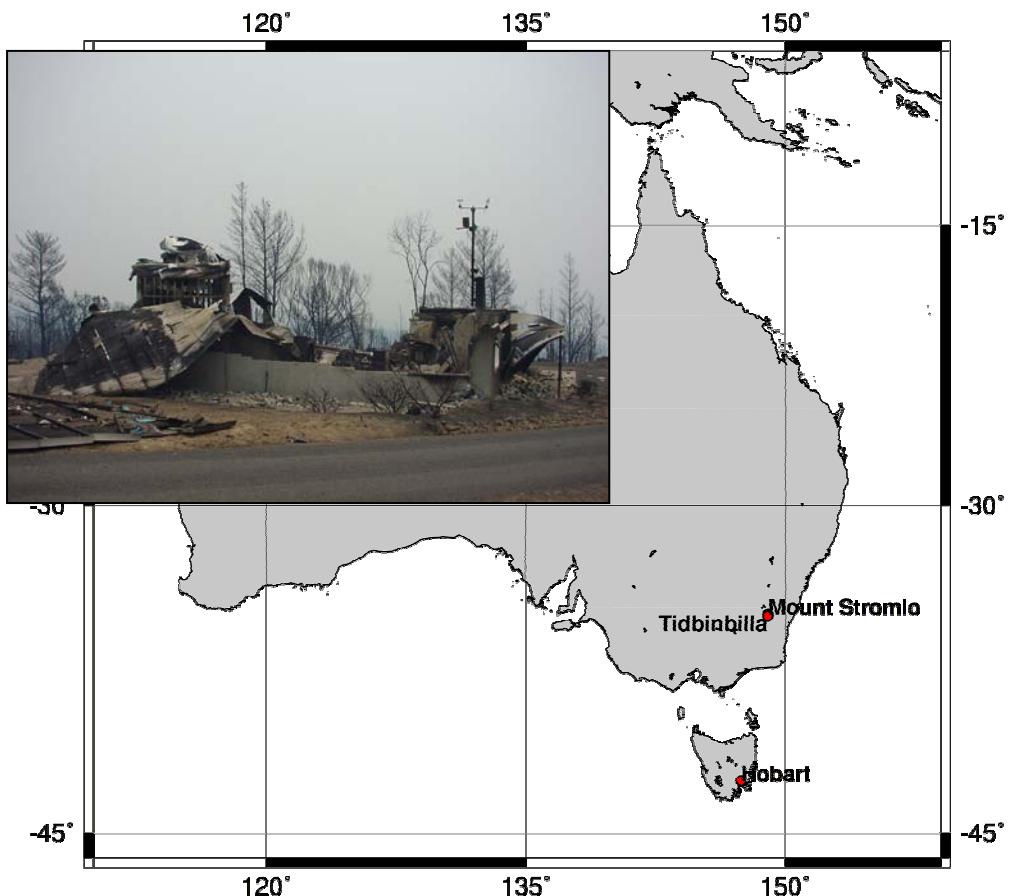
## Local Tie Methodology

The Mount Stromlo local tie survey observation and analysis follows the routine procedure used at Geoscience Australia, it is outlined here for completeness:

- The calibration of all geodetic instrumentation including: total station instruments; levelling staffs; fixed height mounts; and reflectors (targets);
- The observation of a vertical geodetic network by application of geodetic levelling (in our case specifically EDM-Height traversing) to all survey marks in the vicinity of the observatory;
- The observation of a horizontal geodetic network by application of terrestrial geodetic observations, including angles and distances to all survey marks in the vicinity of the observatory;
- The observation of a Global Positioning System (GPS) network on suitable survey marks in the vicinity of the observatory (these marks are included in the geodetic levelling);
- The observation of targets located on the observing system (Satellite Laser Ranging instrumentation) during rotational motion about each of its independent axes. This

includes zenith angle observations to a staff on a levelled survey mark in the vicinity for precise height of instrument determination;

- The reduction of terrestrial geodetic observations, including the correction of observations for instrument and target bias, set reduction and atmospheric effects, and includes the height of instrument determination from observations to a staff;
- Classical geodetic least squares (minimum constraint) adjustment of all terrestrial geodetic observations, including deflection of the vertical and geoid corrections (derived from the Australian national gravimetric geoid, (Johnston and Featherstone, 1998)). This results in terrestrial-only coordinate estimates and their associated variance-covariance matrix (in a local system) of the geodetic network and targets located on the SLR instrumentation;
- Invariant Point (IVP) modelling and estimation, includes the estimation of IVP, the axes of rotation and associated system parameters such as axis orthogonality and the offset of the axes; Includes readjustment of terrestrial-only network;
- Analysis of GPS observations. This results in GPS-only coordinate estimates and associated geocentric variance-covariance matrix;
- Transformation (translation and rotation only) of the readjusted terrestrial network and computed IVP coordinate variance-covariance matrix into a global reference frame including a geocentric variance-covariance matrix (estimated and *a priori*); The previous GPS analysis is used as the global reference frame realisation; and the
- Reduction of the complete solution to stations of primary interest (i.e. those with DOMES numbers) and output of a SINEX format solution file including all *a priori* constraints.



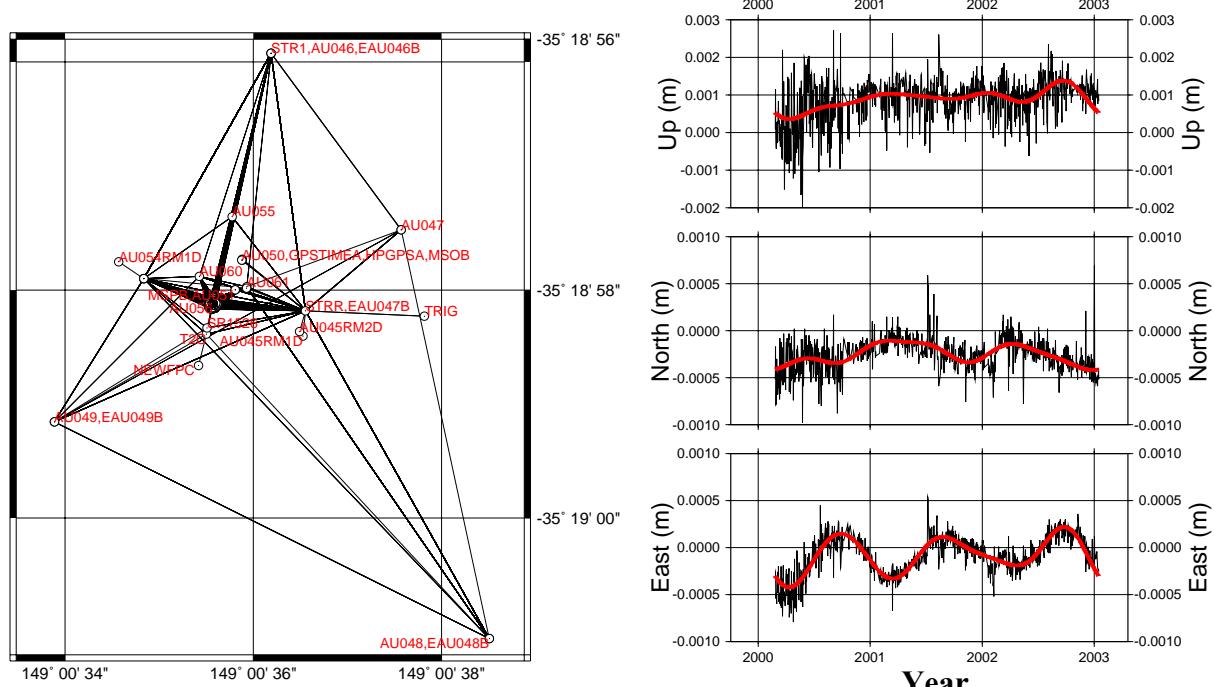
**Figure 1.** Location of the Mount Stromlo Satellite Laser Ranging (SLR) Observatory. The co-located observatory includes SLR, GPS, DORIS and GLONASS systems; Inset: Mount Stromlo Satellite Laser Ranging station post-fire.

## Terrestrial and Global Positioning System (GPS) Observations

Terrestrial observations generally consisted of five sets of observations at each standpoint, see Figure 2. A set consists of a round of face left observations, followed by the reverse round of face right observations. Slope distances and zenith angles were recorded for each observation as well. Atmospheric corrections were not applied in the instrument, but later applied to distances in post processing using conventional correction formulae and local meteorological observations. The heights of instrument were observed using the technique described in Rueger and Brunner (1981), which routinely returns values for height of instrument accurate to 0.1mm. The technique relies on the height difference between the ground marks being determined from levelling observations.

Levelling was carried out using an EDM-height traversing technique. It comprises height difference observations to a prism mounted on a fixed-height prism pole, which is braced by a bi-pole and placed over the survey mark. Differential heighting can then be achieved. This technique minimises thermal expansion effects and refraction caused by thermal flux since the lines of sight are near to parallel along the ground surface.

Long term GPS continuous observations between the STRR (GPS/GLONASS) and STR1 (GPS) stations were used to align the arbitrary local system to the global system, see Figure 3.



**Figure 2.** The Mount Stromlo (1999/2002/2003) terrestrial geodetic network. Terrestrial observations between stations are shown as inter-connecting lines. STRR is the permanent GLONASS station; STR1 is the permanent IGS GPS station.

**Figure 3.** Mount Stromlo GPS baseline time series between STR1 and STRR. Analysis indicates inter-pillar stability at the 1mm level although a sub-millimetre annual signal remains in the baseline (at this time most likely explained by pillar motion).

## Satellite Laser Ranging System Invariant Point Determination

The Mount Stromlo Satellite Laser Ranging (SLR) system reference point is invariant point or IVP and is defined as the intersection of the azimuth axis with the common perpendicular of the azimuth and elevation axes. A method based on 3-dimensional circle fitting is applied as the basis for IVP determination. Three dimensional coordinate observations to targets on the SLR telescope during rotational sequences are used to determine the independent axes of rotation. Multiple realization of the elevation axis (i.e. observed at multiple azimuths) are observed and computed. A least squares method is used for the computation of the axes of rotation and the IVP. A target located on a rigid body, rotating about one independent axis can be fully expressed as a circle in 3-dimensional space. This circle can be described by seven parameters, namely the circle centre (3 parameters), a unit normal vector (3 parameters) perpendicular to the plane of the circle and a circle radius parameter (1 parameter). A minimum of three rotational sequences are required to enable the solution of the equation of a circle.

The method makes the following assumptions: during rotational sequence target paths scribe a perfect circular arc in 3D space; there is no deformation of targeted structure during rotational sequence; there is no axis wobble error; and the axis of interest can be rotated independently of the other axis. No assumptions of axis orthogonality, verticality/horizontality or the precise intersection of the axes are made. The indirect geometrical model includes a number of conditions, including (refer to Figure 4):

- Target paths during rotation about an independent axis scribe a perfect circle in space;
- Circle centres derived from targets observed while being rotated about the same axis are forced to lie along the same line in space;
- Normal vectors to each circle plane derived from targets observed while being rotated about the same axis are forced to be parallel;
- The orthogonality (or non-orthogonality) of the elevation axis to the azimuth axis remains constant over all realisations of the elevation axis;
- Identical targets rotated about a specific realisation of an axis will scribe 3-dimensional circles of equal radius;
- The offset distance between the elevation axis and azimuth axis remains constant over all realisations of the elevation axis;
- The distance between 3-dimensional circle centres for all realizations of the elevation axis are constant over all realisations of the elevation axis; and
- The IVP coordinate estimates remain constant over all realisations (combinations) of the azimuth/elevation axis;

Because the 3-dimensional circle (described by seven parameters) includes a normal vector to the circle plane, the following constraint is also applied;

- The unit normal vector perpendicular to the circle plane is of magnitude one;

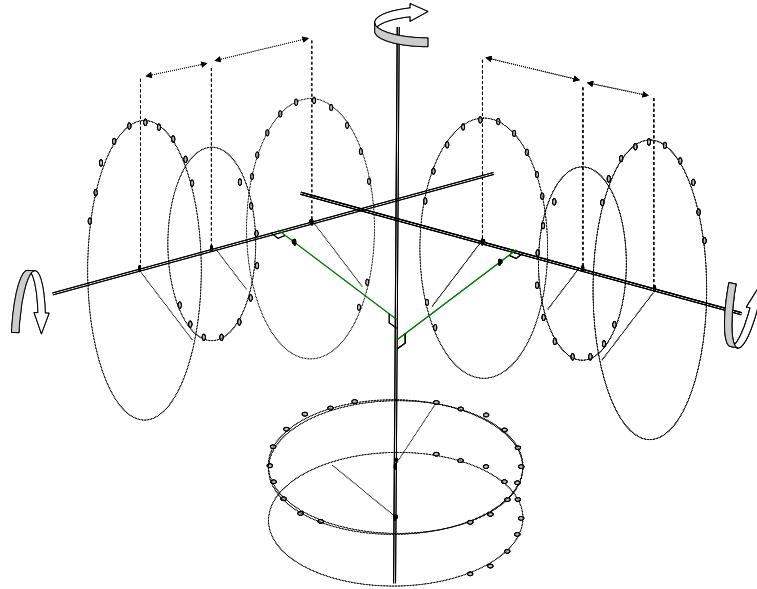
The linearized equations take the form of two sets of equations, namely conditions and constraints with added parameters

$$Av + B\Delta = f$$
$$D_1\Delta + D_2\Delta' = h$$

where  $v$  is the parameter vector of residuals of the input classical adjustment results,  $\Delta$  is the parameter vector of the circle parameters,  $\Delta'$  is the parameter vector of the parameters associated with the IVP estimates,  $f$  and  $h$  are the constant vectors associated with the evaluation of the conditions and constraints respectively and  $A$ ,  $B$ ,  $D_1$  and  $D_2$  are matrixes of coefficients. The least squares solution is obtained from the following system of normal equations (Mikhail, 1976)

$$\begin{bmatrix} -W & A^t & 0 & 0 & 0 \\ A & 0 & B & 0 & 0 \\ 0 & B^t & 0 & D_1^t & 0 \\ 0 & 0 & D_1 & 0 & D_2 \\ 0 & 0 & 0 & D_2^t & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \\ k_c \\ \Delta' \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ h \\ 0 \end{bmatrix}$$

where  $W$  is the weight matrix of the input coordinates derived from the classical adjustment and  $k$  and  $k_c$  are vectors of Lagrange multipliers required to satisfy the Least Squares criteria.



**Figure 4.** Mount Stromlo IVP model. Note that to simplify the diagram only two targets are shown on the azimuth axis and three targets are shown on the elevation axis.

The solution to the normal equation system is iterated as required for the non-linear condition and constraint equations. An updated estimate of the input coordinates and their variance-covariance matrix is obtained together with an estimate of the IVP coordinate, their variance-covariance matrix and the inter-relating covariance matrix.

## Results

Comparisons between the multi-year combined adjustment and each individual survey were undertaken. To remove the influence of datum 3-dimensional translations and a rotation about the local vertical were performed between the combined solution and the individual solutions before comparison. The Root Mean Square (RMS) error between the combined and

1999 coordinate solutions was 0.8, 0.5 and 1.0 mm in the north, east and up components respectively, while the RMS error between combined survey and the 2002 and 2003 surveys was 0.6, 0.2 and 0.3 mm (2002) and 1.2, 0.6, 0.0 mm (2003) in the north, east and up components respectively. Thus in general there is good agreement between the 1999, 2002 and 2003 surveys. Because of this network stability the network realisations have been combined to provide a consistent multi-year local frame at the Mount Stromlo observatory.

Table 1 gives the coordinate of the old SLR invariant point relative to the new laser system. The replacement of axis bearings in the SLR system in late 2001 may account for the small differences between the 1999 determination of **7849 50119S001** to the 2002 determination, namely -0.8, 0.4, -0.6 mm east north up components respectively. Please note that the new SLR system was constructed in approximately the same horizontal position as the old system but approximately 0.95 metres higher.

**Table 1.** IVP determination coordinate relative to SLR IVP 7825 50119S003

	East (mm)	North (mm)	Up (mm)
7849 50119S001 1999 survey	5.1	-1.9	-948.6
7849 50119S001 2002 survey	5.9	-2.3	-948.0
7825 50119S003 2003 survey	0.0	0.0	0.0

The least squares solution of the SLR IVP position included; 36 targets; 5 IVP estimates (constrained together); 1284 pseudo-observations; 252 unknowns; 69 additional unknowns; 846 conditions; 76 constraints and 178 additional constraints. The resultant linear system was 2705 x 2705 with degrees of freedom 2063. IVP model (circle) fit residuals were 0.9 mm Root Mean Square Error (RMS) for the in-plane residuals and 0.7 mm for the out-of-plane residuals. The Root Mean Square Error (RMS) of the terrestrial coordinate observations to the IVP model were 1.1, 0.5 and 0.5 millimetres in the east, north and up components respectively.

## Access to Results

The SINEX file corresponding to this paper is **AUSSTRO0312GB.SNX**, and can be found at <ftp://ftp.ga.gov.au/sgac/sinex/ties/>. This file supersedes both the SINEX file aus00c05.snx submitted to the International Earth Rotation Service (IERS) in 1999 for the ITRF2000 computation and **AUSSTRO0312GA.SNX** which had incorrect time tags.

## References

Johnston G. and W. E. Featherstone, AUSGEOID98: a new gravimetric geoid for Australia, *Australian Surveying and Land Information Group (AUSLIG), available online:* <http://www.ga.gov.au/nmd/geodesy/ausgeoid/docs/iemsgary.pdf>, 1998.

Mikhail E. M., Observations and least squares, Dun-Donnelley Publisher, New York, 1976.

Rueger, J. M. and F.K. Brunner, Practical results of EDM-Height Traversing, *The Australian Surveyor*, June, 1981, Vol. 30, No. 6., 1981.